Hypothesis testing (假設檢定)

- Suppose that we have a sample $(X_1, \ldots, X_n)$ whose distribution is determined by a parameter vector θ. Suppose that we would like to determine whether $\theta \in \Theta^*$ for a given set Θ^*. Then, we can formulate the problem as testing
$$H_0 : \theta \in \Theta^* \text{ versus } H_1 : \theta \notin \Theta^*, \tag{1}$$
or
$$H_0 : \theta \notin \Theta^* \text{ versus } H_1 : \theta \in \Theta^*.$$
Here H_0 is called the null hypothesis (虛無假設) and H_1 is called the alternative hypothesis (對立假設). In hypothesis testing, we control the probability of rejecting H_0 when H_0 is true, which is called the Type I error probability (型一錯誤機率).

- 可樂問題.
  - 正常製程下, 每瓶可樂重量(以公克計)的分布爲 $N(350, 0.4^2)$(近似分布).
  - 假設新產出的可樂每瓶重量分布爲$N(\mu, 0.4^2)$. 想知道: 新產出的可樂平均重量是否未達350公克.
  - Sample: 新產出的 16 瓶可樂重量.

  可樂問題中, 假設新產出的可樂每瓶重量分布爲 $N(\mu, 0.4^2)$. 則可樂平均重量未達350公克代表 $\mu < 350$. 我們可考慮檢定問題
$$H_0 : \mu \geq 350 \text{ versus } H_1 : \mu < 350 \tag{2}$$
或
$$H_0 : \mu < 350 \text{ versus } H_1 : \mu \geq 350. \tag{3}$$
檢定問題要用 (2) 還是 (3)?
  - 如果想控制可樂平均重量未達350公克時,檢定結果卻決定$\mu \geq 350$的機率, 則考慮檢定問題(3).
  - 如果想控制可樂平均重量已達350公克時,檢定結果卻決定$\mu < 350$的機率, 則考慮檢定問題(2).

- Test and test statistic. To decide whether to reject H_0 or not for the testing problem in (1), we need to obtain a sample that is relevant to the testing problem and then make a decision based on some quantity computed from the sample. The decision rule that states when to reject H_0 is called a test (檢定) and the quantity computed from the sample is called a test statistic (檢定統計量).

- 可樂問題. 考慮檢定問題 (3).
  - Sample: 新產出的 16 瓶可樂重量.
  - $\bar{X}$: sample mean.
  - Decision rule: rejecting $H_0 : \mu < 350$ if $\bar{X} > 350 + 0.4 \times 2/\sqrt{16} = 350.2$.

- $\bar{X}$ is called the test statistic in the above testing procedure.

- Once a test is applied, there can be two types of errors.
  - The error of rejecting H_0 when H_0 is true is called Type I error (型一錯誤、型一誤差).
  - The error of failing to reject H_0 when H_0 is false is called Type II error (型二錯誤、型二誤差).

- In general, it can be impossible to require both Type I and Type II error probabilities to be arbitrarily small. Therefore, we only control Type I error probabilities.

- The smallest upper bound for Type I error probabilities is called the size of a test. 可樂問題中, 若$H_0 : \mu < 350$ 且當 $\bar{X} > 350.2$時拒絕H_0, 則 Type I error probability 爲

$$P(N(\mu, 0.4^2/16) > 350.2)$$

  for $\mu < 350$. Since $P(N(\mu, 0.4^2/16) > 350.2)$ increases as μ increases, the smallest upper bound for $P(N(\mu, 0.4^2/16) > 350.2)$ when $\mu < 350$ is

$$\lim_{\mu \to 350^-} P(N(\mu, 0.4^2/16) > 350.2) = P(N(350, 0.4^2/16) > 350.2).$$

  $P(N(350, 0.4^2/16) > 350.2)$ is called the size of the test (the worst Type I error probability).

- Example 1. Suppose that a store places an order of 100 boxes of apples from a farm. The store and the farm has agreed that if the defective rate p is higher than 10%, then all boxes should be returned. They would like to construct a test for determining whether $p > 0.1$, and would like to control the probability that the test rejects $p \leq 0.1$ when $p \leq 0.1$. Which statement should be the null hypothesis H_0, $p \leq 0.1$ or $p > 0.1$?

  Ans. $H_0 : p \leq 0.1$.

- Level of a test. For a given test, if the size of the test $\leq a$, then we say that the test is of level a. That is, for a level a test, the Type I error probabilities are below a:

$$P(\text{ test rejects } H_0) \leq a \text{ when } H_0 \text{ is true.}$$

- 給定H_1的敘述方式. The statement "We can conclude xxx at level a" implies that "xxx" is the alternative hypothesis H_1. 若敘述爲"在顯著水準a之下, 可推論xxx成立", 則表示此時給定的H_1爲xxx成立, 而H_0爲xxx不成立.

  Example 2. Suppose that a store places an order of 100 boxes of apples from a farm. The store and the farm has agreed that if the defective rate p is higher than 10%, then all boxes should be returned. They would like to know whether they can conclude that $p > 0.1$ at level 0.05. Write down H_0 and H_1 for their testing problem.

Ans. $H_0 : p \le 0.1$; $H_1 : p > 0.1$.

- Two-tailed testing problem about μ with known σ. Suppose that $(X_1, \cdots, X_n)$ is a random sample from $N(\mu, \sigma^2)$, where σ is known. Consider the problem of testing

$$H_0 : \mu = \mu_0 \text{ versus } H_1 : \mu \ne \mu_0.$$

Consider the test that rejects H_0 at level a when

$$\left| \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} \right| > z_{a/2},$$

then the test is of size a. The test is called the one sample Z test with $H_1 : \mu \ne \mu_0$.

- Example 3. Desk production example in the text.
  - Desk weekly production $\sim N(200, 16^2)$ based on old data.
  - Assume: weekly production $\sim N(\mu, 16^2)$.
  - Want to decide whether one can conclude that $\mu \ne 200$ at level 0.01.

(a) State the null hypothesis and the alternative hypothesis.

(b) Suppose that the mean number of desks produced per week in last 50 weeks will be used for the testing problem in Part (a). State the decision rule.

(c) Suppose that the mean number of desks produced per week in last 50 weeks was 203.5. Compute the value of the test statistic.

(d) State the decision made.

Answers.

(a) Let μ be the mean number of desks produced per week. The null hypothesis is $H_0 : \mu = 200$ and the alternative hypothesis is $H_1 : \mu \ne 200$.

(b) Decision rule: reject H_0 when

$$\left| \frac{\sqrt{50}(\bar{X} - 200)}{16} \right| > z_{0.01/2} = z_{0.005} \approx 2.576,$$

where $\bar{X}$ is the mean number of desks produced per week for 50 weeks.

(c) The observed value for the test statistic

$$\frac{\sqrt{50}(\bar{X} - 200)}{16}$$

is

$$\frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547.$$

(d) Since $|1.547| < 2.576$, we do not reject H_0 at significance level 0.01.

- Two-tailed testing problem about μ with unknown σ. Suppose that $(X_1, \cdots, X_n)$ is a random sample from $N(\mu, \sigma^2)$, where σ is unknown. Consider the problem of testing

  $$H_0 : \mu = \mu_0 \text{ versus } \mu \neq \mu_0.$$

  Suppose that a test rejects H_0 when

  $$\left| \frac{\sqrt{n}(\bar{X} - \mu_0)}{S} \right| > t_{a/2,n-1},$$

  then the test is of size a. The test is called the one sample t test with $H_1 : \mu \neq \mu_0$. Here S is the sample standard deviation.

  Example 4. In Example 3, suppose that the desk weekly production still follows a normal distribution, but the mean and the standard deviation for the normal distribution are unknown. Suppose that the mean number of desks produced per week in last 50 weeks was 203.5 and the sample standard deviation is 16. Can we conclude that the mean number of desks produced is different from 200 at the 0.01 significance level?

  Sol. We reject $H_0 : \mu = 200$ at level 0.01 when

  $$\left| \frac{\sqrt{50}(\bar{X} - 200)}{S} \right| > t_{0.005,49} \approx 2.680.$$

  Since we observe

  $$\left| \frac{\sqrt{50}(\bar{X} - 200)}{S} \right| = \frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547,$$

  we cannot conclude that the mean number of desks produced is different from 200 at the 0.01 significance level.

- Suppose that $(X_1, \cdots, X_n)$ is a random sample from $N(\mu, \sigma^2)$. Let S be the sample standard deviation. Consider the one-tailed testing problem:

  $$H_0 : \mu \leq \mu_0 \text{ versus } H_1 : \mu > \mu_0.$$

  - If σ is known, consider the test that rejects H_0 at level a if

    $$\frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} > z_a.$$

    It can be shown that the test is of size a (the proof is given in class). The test is called the one sample Z test with $H_1 : \mu > \mu_0$.
  - If σ is unknown, consider the test that rejects H_0 at level a if

    $$\frac{\sqrt{n}(\bar{X} - \mu_0)}{S} > t_{a,n-1}.$$

    It can be shown that the test is of size a (the proof is left as an exercise). The test is called the one sample t test with $H_1 : \mu > \mu_0$.

- Suppose that $(X_1, \cdots, X_n)$ is a random sample from $N(\mu, \sigma^2)$. Let S be the sample standard deviation. Consider the one-tailed testing problem:

$$H_0 : \mu \geq \mu_0 \text{ versus } H_1 : \mu < \mu_0.$$

  - If σ is known, consider the test that rejects H_0 at level a if

$$\frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} < -z_a.$$

    It can be shown that the test is of size a (the proof is left as an exercise). The test is called the one sample Z test with $H_1 : \mu < \mu_0$.
  - If σ is unknown, consider the test that rejects H_0 at level a if

$$\frac{\sqrt{n}(\bar{X} - \mu_0)}{S} < -t_{a,n-1}.$$

    It can be shown that the test is of size a (the proof is left as an exercise). The test is called the one sample t test with $H_1 : \mu < \mu_0$.

- Example 5.

  (a) In Example 3, can we conclude that the mean number of desks produced per week is more than 200 at the 0.01 significance level?

  (b) In Example 3, can we conclude that the mean number of desks produced per week is less than 200 at the 0.01 significance level?

  (c) In Example 4, can we conclude that the mean number of desks produced per week is more than 200 at the 0.01 significance level?

  (d) In Example 4, can we conclude that the mean number of desks produced per week is less than 200 at the 0.01 significance level?

  Sol. Recall that in Example 3 or Example 4, the distribution of the weekly desk production is $N(\mu, \sigma^2)$, and $\bar{X}$ is the sample mean for the number of desks produced per week in 50 weeks.

  (a) The testing problem is

$$H_0 : \mu \leq 200 \text{ v.s. } H_1 : \mu > 200. \tag{4}$$

  Since $\sigma = 16$, consider the test that rejects $H_0 : \mu \leq 200$ at level a when

$$\frac{\bar{X} - 200}{\sqrt{16^2/50}} > z_a.$$

  In Example 3, the observed value for the test statistic $\dfrac{\bar{X} - 200}{\sqrt{16^2/50}}$ is

$$\frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547.$$

  Here $z_a = z_{0.01} = 2.326$. Since $1.547 < 2.326$, we cannot conclude that the mean number of desks produced per week is more than 200 at the 0.01 significance level.

(b) The testing problem is

$$H_0 : \mu \geq 200 \text{ v.s. } H_1 : \mu < 200. \tag{5}$$

Since $\sigma = 16$, consider the test that rejects $H_0 : \mu \geq 200$ at level a when

$$\frac{\bar{X} - 200}{\sqrt{16^2/50}} < -z_a.$$

In Example 3, the observed value for the test statistic $\dfrac{\bar{X} - 200}{\sqrt{16^2/50}}$ is

$$\frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547.$$

Here $z_a = z_{0.01} = 2.326$. Since $1.547 > -2.326$, we cannot conclude that the mean number of desks produced per week is less than 200 at the 0.01 significance level.

(c) The testing problem is given in (4). Since σ is unknown, consider the test that rejects $H_0 : \mu \leq 200$ at level a when

$$\frac{\bar{X} - 200}{\sqrt{S^2/50}} > t_{a,n-1}.$$

In Example 4, the observed value for the test statistic $\dfrac{\bar{X} - 200}{\sqrt{S^2/50}}$ is

$$\frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547.$$

Here $t_{a,n-1} = t_{0.01,49} = 2.405$. Since $1.547 < 2.405$, we cannot conclude that the mean number of desks produced per week is more than 200 at the 0.01 significance level.

(d) The testing problem is given in (5). Since σ is unknown, consider the test that rejects $H_0 : \mu \geq 200$ at level a when

$$\frac{\bar{X} - 200}{\sqrt{S^2/50}} < -t_{a,n-1}.$$

In Example 4, the observed value for the test statistic $\dfrac{\bar{X} - 200}{\sqrt{S^2/50}}$ is

$$\frac{\sqrt{50}(203.5 - 200)}{16} \approx 1.547.$$

Here $t_{a,n-1} = t_{0.01,49} = 2.405$. Since $1.547 > -2.405$, we cannot conclude that the mean number of desks produced per week is less than 200 at the 0.01 significance level.

- p-value. p-value is a number constructed such that both (i) and (ii) hold based on the observed test statistic:

  (i) p-value $< a \Rightarrow$ test rejects H_0 at level a

  (ii) p-value $> a \Rightarrow$ test does not reject H_0 at level a

- For tests such that

$$\text{test rejects } H_0 \text{ at level } a \ \Leftrightarrow \text{xxx} < a,$$

  xxx is the p-value.

- Example 6. Consider the problem of testing

$$H_0 : \mu \geq 200 \text{ versus } H_1 : \mu < 200$$

  based on a random sample from $N(\mu, \sigma^2)$ with $\sigma = 16$. Suppose that a test is proposed and the p-value based on the observed test statistic is 0.02. Based on the test, can we reject H_0 at level a for (a) $a = 0.01$ and (b) $a = 0.05$?

  Answers. (a) No (since $0.01 < 0.02$). (b) Yes (since $0.05 > 0.02$).

- Interpreting the p-value.
  - p-value $< 0.1 \Rightarrow$ some evidence against H_0. We are somewhat confident to conclude H_1.
  - p-value $< 0.05 \Rightarrow$ strong evidence against H_0. We are confident to conclude H_1.
  - p-value $< 0.01 \Rightarrow$ very strong evidence against H_0. We are very confident to conclude H_1.
  - p-value $< 0.001 \Rightarrow$ extremely strong evidence against H_0. We are extremely confident to conclude H_1.

- p-value for two-tailed testing problems about μ with known σ. Suppose that $(X_1, \ldots, X_n)$ is a random sample from $N(\mu, \sigma^2)$. For a test that rejects

$$H_0 : \mu = \mu_0 \text{ versus } \mu \neq \mu_0$$

  at level a when

$$\left| \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} \right| > z_{a/2},$$

  the p-value is

$$2P\left(N(0,1) > \text{ observed } \left| \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma} \right| \right).$$

- Example 7. Consider the problem of testing

$$H_0 : \mu = 200 \text{ versus } H_1 : \mu \neq 200$$

based on a random sample from $N(\mu, \sigma^2)$ with $\sigma = 16$. Consider the test that rejects H_0 at significance level a if

$$\left| \frac{\sqrt{n}(\bar{X} - 200)}{16} \right| > z_{a/2}.$$

Suppose that a random sample of size 50 is collected and the sample mean is 203.5. What is the p-value for this test?

- Solution 1. The observed value for the test statistic $\dfrac{\sqrt{50}(\bar{X} - 200)}{16}$ is 1.55. The p-value for this test is

$$\begin{aligned} 2P(N(0,1) > 1.55) &= 2(0.5 - P(0 < N(0,1) < 1.55)) \\ &\approx 2(0.5 - 0.4394) = 0.1212 \end{aligned}$$

- Solution 2. The observed value for the test statistic $\dfrac{\sqrt{50}(\bar{X} - 200)}{16}$ is 1.55, so we can reject H_0 at level a if

$$|1.55| > z_{a/2}.$$

  Since

$$1.55 > z_{a/2} \Leftrightarrow 2P(N(0,1) > 1.55) < a,$$

  the p-value is

$$\begin{aligned} 2P(N(0,1) > 1.55) &= 2(0.5 - P(0 < N(0,1) < 1.55)) \\ &\approx 2(0.5 - 0.4394) = 0.1212. \end{aligned}$$

- p-value for one-tailed testing problems about μ with known σ. Suppose that $(X_1, \ldots, X_n)$ is a random sample from $N(\mu, \sigma^2)$. Let

$$Z_n = \frac{\sqrt{n}(\bar{X} - \mu_0)}{\sigma}.$$

  - For a test that rejects

$$H_0 : \mu \geq \mu_0 \text{ versus } H_1 : \mu < \mu_0.$$

    at level a when $Z_n < -z_a$, the p-value is $P(N(0,1) <$ observed $Z_n)$.
  - For a test that rejects

$$H_0 : \mu \leq \mu_0 \text{ versus } H_1 : \mu > \mu_0.$$

    at level a if $Z_n > z_a$, the p-value is $P(N(0,1) >$ observed $Z_n)$.

- Example 8. Consider the problem of testing

$$H_0 : \mu \leq 200 \text{ versus } H_1 : \mu > 200$$

  based on a random sample from $N(\mu, \sigma^2)$ with $\sigma = 16$. Consider the test that rejects H_0 at significance level a if

$$\frac{\sqrt{n}(\bar{X} - 200)}{16} > z_a.$$

  Suppose that a random sample of size 50 is collected and the sample mean is 203.5. What is the p-value for this test?

Solution. The observed value for the test statistic $\dfrac{\sqrt{50}(\bar{X}-200)}{16}$ is 1.55. The p-value is

$$P(N(0,1) > 1.55) \approx (0.5 - 0.4394) = 0.0606.$$

- Example 9. Consider the problem of testing

  $$H_0 : \mu \geq 200 \text{ versus } H_1 : \mu < 200$$

  based on a random sample from $N(\mu, \sigma^2)$ with $\sigma = 16$. Consider the test that rejects H_0 at significance level a if

  $$\frac{\sqrt{n}(\bar{X}-200)}{16} < -z_a.$$

  Suppose that a random sample of size 50 is collected and the sample mean is 196.5. What is the p-value for this test?

  Solution. The observed value for the test statistic $\dfrac{\sqrt{50}(\bar{X}-200)}{16}$ is -1.55. The p-value is

  $$P(N(0,1) < -1.55) = P(N(0,1) > 1.55) \approx (0.5 - 0.4394) = 0.0606.$$

- For p-value computation for testing problems involving unknown σ, we need to compute probability involving the t distributions. Let $t(m)$ denote the t distribution with m degrees of freedom. Then

  – `pt(b, df=m)` gives $P(t(m) < b)$.

- Example 10. Consider the problem of testing

  $$H_0 : \mu \leq 200 \text{ versus } H_1 : \mu > 200$$

  based on a random sample from $N(\mu, \sigma)$, where σ is unknown. Suppose that a random sample of size 25 is collected and the sample mean and sample standard deviation are 208 and 16, respectively. Suppose that we will use t test for the above testing problem.

  (a) What can be said about the range of the p-value of t test based on the sample using the table "Quantiles for t distributions"?

  (b) Running the R command

  ```
  1-pt(2.5, df=c(24,25))
  ```

  gives the output

  ```
  [1] 0.009827088 0.009671564
  ```

  Find the p-value of t test based on the sample using the R output.

Sol. The t test rejects H_0 at level a if and only if

$$\Leftrightarrow \frac{\sqrt{25}(208-200)}{16} > t_{a,24},$$

where

$$\frac{\sqrt{25}(208-200)}{16} = 2.5.$$

so the p-value for the test is

$$P(t(24) > 2.5).$$

(a) From the table "Quantiles for t distributions", $t_{0.01,24} < 2.5 < t_{0.005,24}$, so

$$0.005 = P(t(24) > t_{0.005,24}) < P(t(24) > 2.5) < P(t(24) > t_{0.01,24}) = 0.01.$$

The p-value is in the range $(0.005, 0.01)$.

(b) From the R output, $1 - P(t(24) \leq 2.5) = 0.009827088$, so the p-value $P(t(24) > 2.5) = 0.009827088$.

- Testing for population proportion using approximate Z tests. Suppose that $X \sim Bin(n,p)$. Consider testing problems involving comparing p with a given p_0 based on X. Let

$$Z_n = \frac{\sqrt{n}(X/n - p_0)}{\sqrt{p_0(1-p_0)}},$$

then Z_n can be used for the following testing problems.

  - Two-tailed testing problem:

  $$H_0 : p = p_0 \text{ versus } H_1 : p \neq p_0.$$

  Decision rule: reject H_0 at level a when $|Z_n| > z_{a/2}$. p-value: $2P(N(0,1) >$ observed $|Z_n|)$.

  - Right-tailed testing problem:

  $$H_0 : p \leq p_0 \text{ versus } H_1 : p > p_0.$$

  Decision rule: reject H_0 at level a when $Z_n > z_a$. p-value: $P(N(0,1) >$ observed $Z_n)$.

  - Left-tailed testing problem:

  $$H_0 : p \geq p_0 \text{ versus } H_1 : p < p_0.$$

  Decision rule: reject H_0 at level a when $Z_n < -z_a$. p-value: $P(N(0,1) <$ observed $Z_n)$.

  - The above three tests are called the one sample (approximate) Z tests for population proportion.

- For each of the above three tests, the test size is the probability of rejecting H_0 when $p = p_0$, which follows from the fact that $P(Bin(n,p) > C_0)$ increases as p increases for a given pair (n, C_0). Below are R commands for plotting $P(Bin(n,p) > C_0)$ as a function of p on $[0,1]$ for $n = 50$ and $C_0 = 10$.

```
n <- 50
c0 <- 10
pr <- function(p){  1-pbinom(c0, size=n, prob=p) }
curve(pr, 0, 1)
```

- Example 11. Election example in the text.
  - A candidate needs at least 80% of the vote to be elected.
  - 1,550 out of 2,000 sample voters support the candidate.

  Is there any evidence that the candidate will not be elected?

- Solution. Let p be the proportion of voters who would vote for the candidate. Consider the testing procedure that rejects $H_0 : p \geq 0.8$ if

$$\frac{\sqrt{2000}(X/2000 - 0.8)}{\sqrt{0.8(1-0.8)}} < -z_a,$$

  where X is the number of supporters of the candidate in the 2000 sample voters. Then the test is approximately of level a. The observed value for the test statistic $\sqrt{2000}(\bar{X} - 0.8)/\sqrt{0.8(1-0.8)}$ is

$$\frac{\sqrt{2000}(1550/2000 - 0.8)}{\sqrt{0.8(1-0.8)}} \approx -2.80,$$

  so the p-value is

$$P(N(0,1) < -2.80) = 0.5 - 0.4974 = 0.0026 < 0.01.$$

  This is a very strong evidence that the candidate will not be elected ($p < 0.8$).

- The Type II error probability can be calculated for special cases under H_1.
- Example 12. Steel bar example in the text.
  - Historically, tensile strength $\sim N(10000, 400^2)$ (in psi).
  - Assume: tensile strength $\sim N(\mu, 400^2)$.
  - Goal: determine whether $\mu = 10000$.
  - Agreement: accept the incoming lot of shipment if and only if

$$9922 \leq \bar{X} \leq 10078,$$

    where $\bar{X}$ is mean strength for a sample of 100 steel bars.

– Find the probability of failing to reject the shipment when $\mu = 9900$.

- Solution. Let $X_1, \ldots, X_{100}$ be the strengths of the selected 100 bars in the incoming lot. Suppose that $(X_1, \ldots, X_{100})$ is a random sample from $N(\mu, \sigma^2)$. Then the distribution of

$$\frac{\sqrt{100}(\bar{X} - 9900)}{400}$$

is $N(0,1)$ when $\mu = 9900$. In such case, the probability of failing to reject the shipment is

$$\begin{aligned}
&P\left(9922 \le \bar{X} \le 10078\right) \\
&= P\left(\frac{\sqrt{100}(9922 - 9900)}{400} \le \frac{\sqrt{100}(\bar{X} - 9900)}{400} \le \frac{\sqrt{100}(10078 - 9900)}{400}\right) \\
&= P(0.55 < N(0,1) < 4.45) \\
&= P(0 < N(0,1) < 4.45) - P(0 < N(0,1) < 0.55) \\
&\approx 0.5 - 0.2088 = 0.2912.
\end{aligned}$$

Note that we cannot find $P(0 < N(0,1) < 4.45)$ using the normal table on the course website, but from the table, $P(0 < N(0,1) < 3.09) = 0.4990$, so if we approximate $P(0 < N(0,1) < 4.45)$ using 0.5, the approximation error is less than 0.001.

- In the Steel bar example, let p_1 be the probability of failing to reject the shipment. Below are the p_1 values for different μ's. Can you see any pattern(s)?

	μ =9900	μ =9800	μ =9700	μ =9600	μ =9500
p_1	2.91×10^{-1}	1.14×10^{-3}	1.43×10^{-8}	4.44×10^{-16}	0

Ans. p_1 decreases as μ decreases from 9900 to 9500 (moving further away from the target value 10000).